Linear Time-Varying Model Predictive Control for Automated Vehicles: Feasibility and Stability under Emergency Lane Change

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Introduction

Automated driving is approaching

- enabled by a chain of complex functional modules;
- reliable steering control is essential for operational safety.

Model predictive control (MPC) is promising

- provides a systematic approach to handle (possibly time-varying) constraints;
- exhibits reliable performance in practice.

Potential pitfalls

The certification of feasibility and stability for emergent situations, particularly when interfacing with other functional modules like path planner.

Preliminaries (1): The multi-model MPC approach

- Discrete-time nonlinear systems:

$$z(k+1) = f(z(k), u(k)) + \omega(k), \tag{1}$$
$$z(k) \in \mathcal{Z},\ u(k) \in \mathcal{U},\ \Delta u(k) = u(k) - u(k-1) \in \Delta\mathcal{U},\ \forall k \in \mathbb{N}_+ \tag{2}$$

  where $\omega(k) \in \mathbb{R}^n$ are undesired change introduced by other subsystems.

- The multi-model approach replace the nonlinear system with the model set

$$\Gamma = \{(A, B) \in \mathbb{R}^{m\times m} \times \mathbb{R}^{m\times n} :\ A = A(\xi), B = B(\xi), \xi \in \Xi\}. \tag{3}$$

  with $\xi = [z_r^T\ u_r^T]^T$ being reference state and control, $A(\xi)$, $B(\xi)$ as linearized model, and the set

$$\Xi = \{\xi \in \mathbb{R}^{m+n} : \xi_{\text{min}} \leq \xi \leq \xi_{\text{max}}\}$$

  including all possible combination of z_r, u_r up to the mesh resolution.

Preliminaries (2): The multi-model MPC approach

Assuming $\omega(k)$ absent, given reference path and control $\{z_r(k), z_r(k+1), ...\}$ $\{u_r(k), u_r(k+1), ...\}$, the system equation is

$$\tilde{z}(k+1) = A(\xi(k))\tilde{z}(k) + B(\xi(k))\tilde{u}(k) \tag{4}$$

where $\tilde{z}(k) = z(k) - z_r(k)$ and $\tilde{u}(k) = u(k) - u_r(k)$. MPC solves the problem

$$\min_{\tilde{U}_t} \quad J(t) = \tilde{z}_{t+N|t}^T Q_f \tilde{z}_{t+N|t} + \sum_{k=t}^{t+N-1} \tilde{z}_{k|t}^T Q \tilde{z}_{k|t} + \tilde{u}_{k|t}^T R \tilde{u}_{k|t} \tag{5a}$$

$$\text{s. t.} \quad \text{(2), (4), initial state constraint, and final state constraint } \tilde{\mathcal{Z}}_f \tag{5b}$$

Preliminaries (3): Previous work

Assume no mismatch between (1) and (4). For all $\xi(t+N+1) \in \Xi$:

- The feasibility condition shall hold: starting from $z(t+1)$, (5) shall be feasible, which depends on $\tilde{\mathcal{Z}}_f$;
- The stability condition shall hold: $J(t+1) < J(t)$, which depends on Q_f.

Previous work[1]:

- Applied as final constraint the invariant set $\bar{\mathcal{O}}_\infty^{\mathrm{LQR}}$ for all $\xi \in \Xi$ under the corresponding LQR control:

$$L_{\mathrm{LQR}}(\xi) \in \arg\min_{L \in \mathbb{R}^{n \times m}} \sum_{k=0}^{\infty} \tilde{z}(k)^T Q \tilde{z}(k) + \tilde{u}(k)^T R \tilde{u}(k), \text{ where } \tilde{u}(k) = L\tilde{z}(k) \implies$$

$$A(\xi)^T \Big(P(\xi) - P(\xi)B(\xi)\big(B(\xi)^T P(\xi) B(\xi) + R\big)^{-1} B(\xi)P(\xi) \Big) A(\xi) + Q - P(\xi) = 0;$$

- Introduced a design procedure, providing candidates Q_f fulfilling necessary conditions of stability.

[1] P.F. Lima, *et al.* Experimental validation of model predictive control stability for autonomous driving. *Control Engineering Practice*, 81, 244–255, 2018.

Contributions (1): Feasibility condition

When can $\omega(k)$ be present and how large can it be?

Given $\tilde{z}(k) \in \mathcal{Z}_w$, where $\mathcal{Z}_w$ is some specified range for states, the $\omega(k)$ introduced by the interfacing system should be within $\mathcal{W}$ where

$$\mathcal{W} \oplus \mathcal{Z}_w = \bar{\mathcal{O}}_\infty^{\mathrm{LQR}},$$

with $\oplus$ denoting the Minkowski addition.
The feasibility can be ensured provided that

$$\mathcal{W} \oplus \mathcal{Z}_w = \mathcal{K}_N(\bar{\mathcal{O}}_\infty^{\mathrm{LQR}}),$$

with $\mathcal{K}_N(\cdot)$ denoting N step reachable set, which is computationally intractable, while $\bar{\mathcal{O}}_\infty^{\mathrm{LQR}}$ can be computed and $\bar{\mathcal{O}}_\infty^{\mathrm{LQR}} \subset \mathcal{K}_N(\bar{\mathcal{O}}_\infty^{\mathrm{LQR}})$.

Contributions (2): Stability condition

What suffices for Q_f to ensure stability?

Given $\tilde{\mathcal{Z}}_f = \bar{\mathcal{O}}_\infty^{\mathrm{LQR}}$, stability condition is equivalent to

$$A_{\mathrm{cl}}(\xi)^T Q_f A_{\mathrm{cl}}(\xi) + L_{\mathrm{LQR}}(\xi)^T R L_{\mathrm{LQR}}(\xi) + Q - Q_f \preceq 0, \tag{6}$$

where $A_{\mathrm{cl}}(\xi) = \big(A(\xi) + B(\xi)L_{\mathrm{LQR}}(\xi)\big)$. This in turn is equivalent to linear matrix inequalities (LMIs)

$$\begin{bmatrix} Q_f & Q_f A_{\mathrm{cl}}(\xi) \\ A_{\mathrm{cl}}(\xi)^T Q_f & Q_f + \Delta V(\xi) \end{bmatrix} \succeq 0, \ \forall \xi \in \Xi, \tag{7}$$

where $\Delta V(\xi)$ is given as

$$\Delta V(\xi) = A_{\mathrm{cl}}(\xi)^T P(\xi) A_{\mathrm{cl}}(\xi) - P(\xi), \tag{8}$$

where $P(\xi)$ is given by the solution of the algebraic Riccati equation for the system (4) for a specific $\xi \in \Xi$.

Automated vehicle application (1)

The emergency lane change (ELC) senario of consideration:

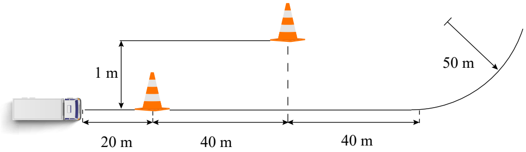


Figure: The scenario for LTV-MPC controller stability test.

Kinematic bicycle model in road aligned frame is used for MPC control:

$$
\begin{aligned}
e_y' &= \frac{\rho_s - e_y}{\rho_s}\tan(e_\psi),\\
e_\psi' &= \frac{(\rho_s - e_y)}{\rho_s \cos(e_\psi)}\kappa - \psi_s'.
\end{aligned}
\quad (9)
$$

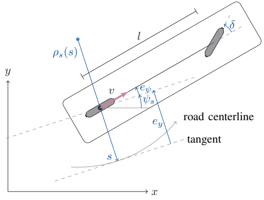


Figure: A nonlinear bicycle model in the road aligned framework.

Automated vehicle application (2)

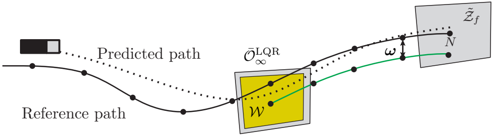


Figure: An ELC is planned to occur at $k = 5$ where the changed lane is in green and the set $\mathcal{W}$ in yellow. In this particular illustration, such ELC would be regarded as feasible since $\tilde{z}(5) + \omega(5) \in \bar{\mathcal{O}}_\infty^{\mathrm{LQR}}$.

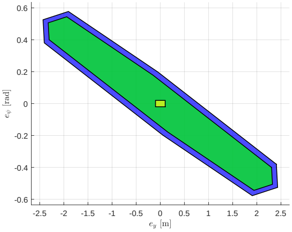


Figure: Illustration of $\mathcal{Z}_w$, $\mathcal{W}$, and $\bar{\mathcal{O}}_\infty^{\mathrm{LQR}}$, where $\mathcal{W}$ is obtained via (6).

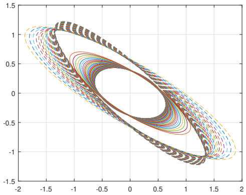
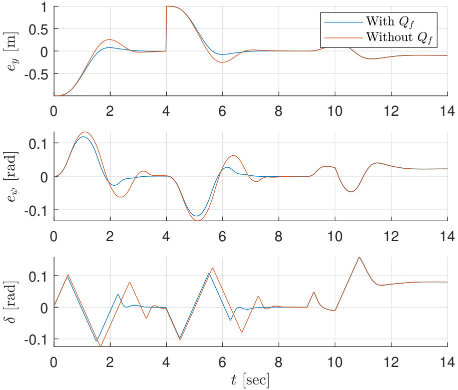
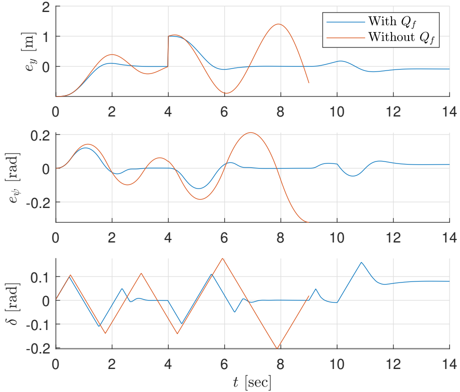


Figure: The ellipses of of $x^T\big(Q_f - P(\xi)\big)x = 1$ and $x^T A_{\mathrm{cl}}(\xi)^T\big(Q_f - P(\xi)\big)A_{\mathrm{cl}}(\xi)x = 1$ with different ξ.

Automated vehicle application (3)



(a) States and control with and without final stage costs where $Q_{11} = 2$.



(b) States and control with and without final stage costs where $Q_{11} = 3$.

Figure: Closed-loop system behavior under different state costs Q.

Conclusion

- Device the lateral control of automated vehicles as an LTV-MPC problem;
- Cast the computation of final cost as LMIs;
- The feasibility of the LTV-MPC under setpoint changes is ensured by introducing a bound on the change magnitudes;
- An emergency lane change scenario is used to demonstrate the feasibility and stability analysis of LTV-MPC under setpoint changes.